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CONVECTIVE HEAT TRANSFER IN HORIZONTAL CYLINDER

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Abstract—A stationary heat crossflow through a horizontal cylindrical cavity filled with water has been investigated depending on the flow direction in a gravity field in the interval of Grashof's numbers ranging from 15 to 5×10^6.

Résumé—Le flux de chaleur, en régime permanent, dans une cavité cylindrique horizontale remplie d'eau a été étudié, en fonction de la direction de l'écoulement, dans un champ de gravité correspondant à des nombres de Grashof variant de 15 à $5 \cdot 10^6$.

Zusammenfassung—Die Abhängigkeit des stationären Wärmestroms von seiner Richtung im Schwerefeld wurde an einem waagrechten, zylindrischen und mit Wasser gefüllten Hohlraum untersucht. Der Bereich der Grashofzahlen erstreckte sich dabei von 15 bis 5×10^6.

Аннотация—Винтервале чисел Грассхофа от 15 до $5{,}10^6$ исследована зависимость стационарного поперечного теплопотока через заполненную водой горизонтальную цилиндрическую полость от его направления в поле тяжести.

THE study of convective heat transfer through cavities filled either with a liquid or a gas under various heating conditions is of a special interest for numerous practical and geophysical problems. In spite of the fact that there are some indications in the literature [1–4], the problem as a whole has not yet been worked out sufficiently. Especially we have little data about the dependence of heat conduction on the orientation of heat delivery surfaces in a gravity field. This refers however not only to convective heat transfer in cavities but also to heat transfer through interlayers, as well as to a number of external problems of convection [5, 6]. A stationary heat crossflow through a horizontal cylindrical cavity filled with water was investigated experimentally depending on the flow direction towards the vector of gravity acceleration. The results of the investigation are presented in this paper.

The experiments were carried out on two models, one large and one small. The hollow cylinder made of polymethyl metacrilate AKP-7 of 70·3 and 100·2 mm in diameter and 210 mm in length was the essential part of the larger model. The wires of thin differential copper–constantan thermocouples were bricked up in the cylinder along its generating lines. Eight hot junctions of thermocouples were soldered to small brass plates pressed at the same level with the internal surface at the centre of the cylinder and at equal distances from each other along the perimeter. Eight thermocouples were mounted in the same way on its external surface. The cylinder was tightly fixed in a round channel drilled along the longer axis of a steel rectangular parallelepiped of $210 \times 135 \times 135$ mm in size and then the cylinder was hermetically covered from its front surfaces by glass and filled with distilled water. An electrical heater and a jet cooler were attached to two opposite edges of the parallelepiped producing a heat flow normal to the axis of the cylinder. The model could turn round the horizontal axis, coinciding with the longitudinal axis of a cavity.

The small model cavity of 9·11 mm in diameter and 80 mm in length was drilled in a Plexiglas block of $30 \times 80 \times 80$ mm in size. The wires of eight copper–constantan thermocouples (the wires were of 0·03 mm in diameter) were pasted into thin longitudinal grooves of the cavity wall in such a way that thermocouple junctions were located at equal distances along the perimeter in the centre of the cavity. The uneveness of the wall was carefully abraded. The other eight hot junctions were established equidistantly in thin channels, drilled 40 mm deep parallel to the axis of the cavity and 11·7 mm from it. The total cold junction of all thermocouples was established on the cavity wall, and its temperature was controlled by a special thermocouple. In other respects the smaller model is analogous to the larger one.

The supply of both the heater and cooler of the installation was stable. The beginning of a stationary temperature régime (time of establishment was equal to 8–16 hr) was controlled at the end of each experiment by records of all the thermocouples taken three times in 30 min.

Photo-registration of the character of motion of liquid by the method of optical lattice as well as by visual observations was carried out in addition to the temperature measurements, as was done in [7].

The treatment of data was made by the method of harmonic analysis. It was found that the temperature distribution on the cavity surface and on the cylindrical surface coaxial to the cavity inside the mass in any direction of the heat flow is described by equations (1) and (1′)

$$T = a_0 + a_1 \cos \varphi \quad (1)$$

$$T' = a_0 + a_1' \cos \varphi' \quad (1')$$

with an accuracy of up to 3–8 per cent.

a is the coefficient of the Fourier series; a_0 is identical with the temperature, averaged according to the volume of the cavity; a_1 and a_1' are the temperatures of the hottest part of the cavity surface, and of the cylindrical surface inside the mass respectively (heat poles), which were measured from the temperature a_0; φ and φ' are the angles measured from the heat poles. Equations (1) and (1′) were used as boundary conditions for the evaluation of heat flows by the solution of the stationary heat conduction equation, for the cylindrical layer involving the cavity.

The relations of molecular heat conductivities of both the water and the wall of the cavity, calculated on the basis of the experimental results at such a position of a model (upper heating) when no convection exists—approximately at the room temperatures—appeared to be equal to 3·7 ± 0·2 and 3·1 ± 0·2 respectively for the larger model polymer of (CH_2 CH—COOH)$_n$, and the smaller one (Plexiglas). It should be noted here, that the ratio of water and Plexiglas heat conductivities at 20°C is equal to 3·26 [8].

The local Nusselt number was evaluated according to the formula

$$Nu = -\frac{\pi R \lambda_e}{2\, a_1 \lambda} \left(\frac{\partial T_e}{\partial r}\right)_R \quad (2)$$

where R is the radius of the cavity, λ_e/λ is the ratio of molecular heat conductivities of the mass and liquid obtained experimentally, $(\partial T_e/\partial r)_R$ is the radial gradient of temperature in a mass at the boundary of a cavity, which can be calculated by solution of the heat conduction equation for the wall of the cavity. It was found that the field of the Nusselt number on the surface of a cavity might be determined by

$$Nu = Nu_0 \cos \psi$$

where Nu_0 is a maximum value of the Nusselt number (the pole situated against the heater), ψ is an angle, which may be measured from this pole. It should be noted that an angular divergence approaching 60° at the side heating arises between identical poles of the Nusselt number and temperature, as a result of the convective heat transfer directed upwards.

The experimental data on heat transfer were generalized in the form of the dependence suggested by Zhukhovitskii [9]. It appeared, that

$$\overline{Nu} = C \left(\frac{Gr\,.\,Pr^2}{1 + Pr}\right)^{1/4} = C\, K^{1/4} \quad (3)$$

where $\overline{Nu} = (2/\pi)\, Nu_0$ is the Nusselt number averaged over the hot (cold) wall of the cavity. The coefficients in equation (2) were choosen in such a way that $\overline{Nu}$ coincides in number with the ratio of effective (convective plus molecular), and pure molecular heat conductivities of the cavity. At the evaluation of the Grashof (Gr) and Prandtl (Pr) numbers the parameters of liquid were taken for the temperature averaged over the cavity volume; both the radius of a cavity and temperature a_1 of the heat pole of the

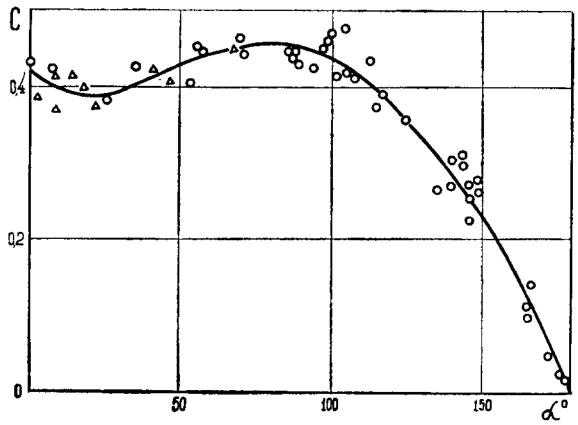

FIG. 1. The variation of C with angle.
○–strict laminar motion.
△–the boundary layer is unstable.

Table 1. *The range of investigated values of K*

$\alpha°$	0–110	130	150	160	170	175
K_{min}	$1{\cdot}6 \times 10^2$	$2{\cdot}5 \times 10^2$	8×10^2	3×10^3	3×10^4	$4{\cdot}10^5$
K_{max}	$2{\cdot}5 \times 10^6$	$2{\cdot}5 \times 10^6$	4×10^6	5×10^6	10^7	$2{\cdot}10^7$

cavity taken from the mean temperature of the liquid, were taken for the characteristic dimension and characteristic temperature difference. The Prandtl number in the experiments varied from 3·9 to 9·5. The coefficient C depends on the orientation of a model in the gravity field.

Figure 1 represents the values of $C = \overline{Nu}/K^{1/4}$ as a function of an angular distance α between the vector of gravity acceleration and the heat pole on the surface of the cavity ($\alpha = 180°$ at the upper heating). Fig. 1 shows that at $\alpha < 90°$ the values of C vary non-uniformly with the angle and reach the maximum at $\alpha \approx 90°$. The drop of C for $\alpha > 90°$ may be explained by the rapid decrease of the intensity of the convective liquid motion which vanishes at $\alpha = 180°$.

Table 1 gives the range of the investigated values of K for various angles. It was found by the optical method of investigation that a boundary layer will be formed at the cavity walls at $K > K_{min}$; while at $K < K_{min}$ boundary layers at hot and cold walls of the cavity are closing and convection is going on without formation of a boundary layer. Moreover the appearance of a stronger heat transfer law is outlined, which is apparently quadratic about K [4, 10]. Therefore the minimum values of K (Table 1), determine the lower limit of the application of (3). For the angles of $\alpha > 65°$ at the maximum values of K the instability of a boundary layer [7] was observed though it never transferred into the developed turbulence. For such a régime of motion equation (3) appeared to be valid as well. Thus, equation (3) describes the heat transfer both by a strict laminar layer and by a hardly disturbed boundary layer. Since the stability of a boundary layer sharply increases with the angle [7], then at $\alpha > 65°$ one may expect the same sharp increase of the upper limits of K up to the values essentially exceeding the investigated ones.

REFERENCES

1. G. A. OSTROUMOV, *Svobodnaya Konvektsiya v Usloviyakh Vnutrennei Zadachi*, GITTL, M.-L. (1952).
2. V. A. TETYUEV, *Uchenye Zapiski Permskogo Gosuniversiteta*, **15**, No. 4, 59 (1958).
3. G. F. SHAIDUROV, *Zh. Tekh. Fiz.* **28**, 855 (1958).
4. G. Z. GERSHUNI and E. M. ZHUKHOVITSKII, *Dokl. Akad. Nauk*, **124**, 289 (1959).
5. M. P. ENIKEEV, *Trudy Kazanskogo Sel'khozinstituta*, **B 39**, 37 (1958).
6. G. F. SHAIDUROV, *Inzh.-Fiz. Zh.* **2**, No. 12, 68 (1959).
7. V. S. CHIRKIN, *Teplofizicheskie Svoistva Materialov*, M., Fizmatgiz (1959).
8. S. S. KUTATELADZE, V. M. BORISHANSKII, I. I. NOVIKOV and O. S. FEDYNSKII, *Zhidkometallicheskie Teplonositeli*, M., Atomizdat (1958).
9. E. M. ZHUKHOVITSKII, *Zh. Tekh. Fiz.* **22**, 832 (1952).
10. G. GRÖBER, S. ERK and U. GRIGULL, *Wärmeübertragung* (in Russian), IL., M. (1958).